First-order formalism for the quintom model of dark energy

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Abstract

The present paper deals to the quintom model of dark energy. We introduce a first-order formalism, which shows how to relate the potential that specifies the scalar field model to Hubble parameter. Reviewing briefly the quintom scenario of dark energy, we present a general procedure to solve the equations of motion for quintom model driven by a couple scalar fields with first-order differential equations.

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1 Introduction

Recent observations from type Ia supernovae [1] in associated with Large Scale Structure [2] and Cosmic Microwave Background anisotropies [3] have provided main evidence for the cosmic acceleration. The combined analysis of cosmological observations suggests that the universe consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although the nature and origin of dark energy are unknown, we still can propose some candidates to describe it. The most obvious theoretical candidate of dark energy is the cosmological constant λ (or vacuum energy) [4, 5] which has the equation of state $w = -1$. However, as is well known, there are two difficulties arise from the cosmological constant scenario, namely the two famous cosmological constant problems — the "fine-tuning" problem and the "cosmic coincidence" problem [6]. An alternative proposal for dark energy is the dynamical dark energy scenario. The dynamical dark energy proposal is often realized by some scalar field mechanism which suggests that the energy form with negative pressure is provided by a scalar field evolving down a proper potential.

So far, a large class of scalar-field dark energy models have been studied, including quintessence [7], K-essence [8], tachyon [9], phantom [10], ghost condensate [11] and quintom [12], and so forth. In addition, other proposals on dark energy include interacting dark energy models [13], Chaplygin gas models [14],holographic dark energy [15], and many others. Recently there are many relevant studies on phantom energy [16]. The analysis of the properties of dark energy from recent observations mildly favor models with w crossing -1 in the near past. But, neither quintessence nor phantom can fulfill this transition. In the quintessence model, the equation of state $w = p/\rho$ is always in the range $-1 \leq w \leq 1$ for $V(\phi) > 0$. Meanwhile for the phantom which has the opposite sign of the kinetic term compared with the quintessence in the Lagrangian, one always has $w \leq -1$. Neither the quintessence nor the phantom alone can fulfill the transition from $w > -1$ to $w < -1$ and vice versa. Although for k-essence[8] one can have both $w \geq -1$ and $w < -1$, it has been lately considered by Ref[17, 18] that it is very difficult for k-essence to get w across -1 during evolving. But one can show [12, 19] that considering the combination of quintessence and phantom in a joint model, the transition can be fulfilled. This model, dubbed quintom, can produce a better fit to the data than more familiar models with $w \geq -1$. In the other term the quintom model of dark energy represents a transition of dark energy equation of state from $w > -1$ to $w < -1$, or vice versa, namely from $w < -1$ to $w > -1$ is also one realization of quintom, as can be seen clearly in [20].

In this paper we focus attention on the quintom model of dark energy by first-order formalism [21](see also [22]), which shows how to relate the potential that specifies the scalar field model to Hubbles parameter. By using [21] we can continue this process for the two fields in quintom model, in another term we present a general procedure to solve the equations of motion for quintom model driven by a couple scalar fields with first-order differential equations.

2 The quintom model of dark energy

The quintom model of dark energy [19] is of new models proposed to explain the new astrophysical data, due to transition from $w > -1$ to $w < -1$, i.e. transition from quintessence dominated universe to phantom dominated universe. Here we consider the spatially flat Friedman-Robertson-Walker universe, where has following space-time metric,

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2). \tag{1}$$

Containing the normal scalar field σ and negative kinetic scalar field ϕ, the action which describes the quintom model is expressed as the following form,

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{4} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)\right), \tag{2}$$

where we have not considered the lagrangian density of matter field and we are using $4\pi G = 1$. In the spatially flat Friedman-Robertson-Walker (FRW) universe, the effective energy density, ρ, and the effective pressure, P, of the scalar fields can be described by;

$$\rho = -\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\sigma}^2 + V(\phi,\sigma), \tag{3}$$

$$P = -\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\sigma}^2 - V(\phi,\sigma). \tag{4}$$

So, the equation of state can be written as,

$$w = \frac{-\dot{\phi}^2 + \dot{\sigma}^2 - 2V(\phi,\sigma)}{-\dot{\phi}^2 + \dot{\sigma}^2 + 2V(\phi,\sigma)}. \tag{5}$$

From the equation of state , it is seen that for $\dot{\sigma} > \dot{\phi}$, $w \geq -1$ and for $\dot{\sigma} < \dot{\phi}$, we will have, $w < -1$. So, the evolution equation for two scalar fields in FRW model will have the following form,

$$\ddot{\phi} + 3H\dot{\phi} - \frac{dV(\phi)}{d\phi} = 0, \tag{6}$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{dV(\sigma)}{d\sigma} = 0,, \tag{7}$$

where, H is the Hubble parameter, $H \equiv \dot{a}/a$. The first Friedmann equation is given by,

$$H^2 = \frac{2}{3}\rho. \tag{8}$$

Substitute ρ into above equation we obtain

$$H^2 = \frac{1}{3}\left[-\dot{\phi}^2 + \dot{\sigma}^2 + 2V(\phi,\sigma)\right], \tag{9}$$

$$\dot{H} = (\dot{\phi}^2 - \dot{\sigma}^2). \tag{10}$$

3 First-order formalism

Now we introduce a first-order formalism, which shows how to relate the potential that specifies the scalar field model to Hubbles parameter. In order to obtain the first-order equation, we use [21]

$$H = W, \qquad \dot{\phi} = +W_{\phi}, \qquad \dot{\sigma} = -W_{\sigma}. \tag{11}$$

From Eqs.(9), (11) the explicit form of the potential is

$$V(\phi, \sigma) = \frac{3}{2}W^2 + \frac{1}{2}(W_{\phi}^2 - W_{\sigma}^2), \tag{12}$$

where the super - potential W is a well behaved function in the space of scalar fields $\phi(x,t)$ and $\sigma(x,t) \in Maps(R^{1,1}, R^2)$. Here we assume that the superpotential be additive as $W(\phi, \sigma) = W_1(\phi) + W_2(\sigma)$, so we have,

$$W_{\phi\sigma} = W_{\sigma\phi} = 0. \tag{13}$$

The equations (11) are first-order differential equations, and they consistent with the set of equations (6, 7) and (9, 10) for the potential (12). The constraint (13) guide us to consider a super-potential with the following form [23, 24]:

$$W(\phi, \sigma) = \sinh K_{\phi}\phi + \sin K_{\sigma}\sigma \tag{14}$$

where K_{ϕ} and K_{σ} are constant. Using Eq.(12), we get the following potential,

$$V(\phi, \sigma) = \frac{3}{2}[\sin(K_{\sigma}\sigma) + \sinh(K_{\phi}\phi)]^2 + \frac{1}{2}(K_{\phi}^2 \cosh^2(K_{\phi}\phi) - K_{\sigma}^2 cos^2(K_{\sigma}\sigma)). \tag{15}$$

Now to obtain ϕ and σ in term of t, we use equation (11)

$$\frac{d\phi}{dt} = W_{\phi} = K_{\phi}\cosh(K_{\phi}\phi), \tag{16}$$

$$\frac{d\sigma}{dt} = -W_{\sigma} = -K_{\sigma}\cos(K_{\sigma}\sigma). \tag{17}$$

Now we are going to obtain $\phi(t)$ and $\sigma(t)$,

$$\phi(t) = \frac{1}{K_{\phi}} \ln\left[\tan(\frac{{K_{\phi}}^2 t}{2})\right], \tag{18}$$

and

$$\sigma(t) = \frac{1}{K_{\sigma}} \sin^{-1}\left[\tanh(-{K_{\sigma}}^2 t)\right]. \tag{19}$$

Using the above equations, we can rewrite the super-potential (14) as function of t ,

$$W(t) = H(t) = \sinh\left[\ln(\tan\frac{{K_{\phi}}^2 t}{2})\right] + \tanh(-K_{\sigma}^2 t). \tag{20}$$

As we know $W(t) = H(t)$, then we can obtain the energy density as a fact in of time,

$$\rho(t) = \frac{3}{2}H^2(t) = \frac{3}{2}W^2(t) = \frac{3}{2}(\sinh\left[\ln(\tan\frac{{K_\phi}^2 t}{2})\right] + \tanh(-K_\sigma^2 t))^2. \tag{21}$$

Also from Eqs.(4, 18, 19) one can obtain the pressure as function of time :

$$\begin{aligned} P(t) &= -\frac{3}{2}W^2(t) + \left[W_\sigma^2(t) - W_\phi^2(t)\right] = K_\sigma^2\cos^2(K_\sigma\sigma) - K_\phi^2\cosh^2(K_\phi\phi) \\ &- \frac{3}{2}\left[\sinh^2(K_\phi\phi) + \sin^2(K_\sigma\sigma) + 2\sinh(K_\phi\phi)\sinh(K_\sigma\sigma)\right]. \end{aligned} \tag{22}$$

Now we are going to write the equation of state as follow,

$$\omega = \frac{P(t)}{\rho(t)} = -1 + \frac{2(W_\sigma^2 - W_\phi^2)}{3W^2}. \tag{23}$$

and the acceleration parameter $q(t)$ given by,

$$q(t) = 1 + \frac{\dot{H}}{H^2} = 1 + \frac{W_\phi^2 - W_\sigma^2}{W^2} \tag{24}$$

In Figure 1 we plot $H(t)$, $\omega(t)$ and $q(t)$ for some choice of parameters.
Substituting Eqs.(18), and (19) respectively in Eqs.(16), (17) one can obtain

$$W_\phi = \frac{K_\phi(tan^2(\frac{K_\phi^2 t}{2}) + 1)}{2tan(\frac{K_\phi^2 t}{2})} \tag{25}$$

$$W_\sigma = K_\sigma\sqrt{1 - tanh^2(K_\sigma^2 t)} \tag{26}$$

If $W_\phi^2 < W_\sigma^2$, then $\omega > -1$, in this case we are in quintessence phase, in the other hand if $W_\phi^2 > W_\sigma^2$, then $\omega < -1$ in this case the universe is in the phantom phase.
Therefore we have extended the first-order formalism introduced in [21, 22, 26] to describe the FRW cosmology, driven by a couple of scalar fields σ, ϕ with standard dynamics for flat spatial geometry. The present method may be used to investigate several interesting cases, in particular the case in which the cosmic evolution occurs in closed or open geometry, for phantom or quintom models.

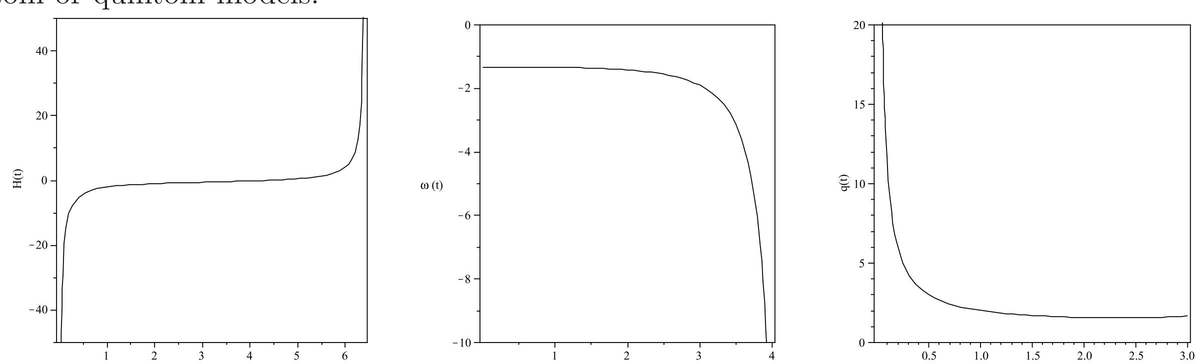

Figure 1: The Graphs of $H(t)$, $\omega(t)$ and $q(t)$ draw in term of t for $K_\phi = 0.7$ and $K_\sigma = 0.4$.

4 conclusion

The idea to consider the Hubble parameter as a function of scalar fields and to transform Eqs.(6,7,9,10) into Eqs.(11,12) has been used in the HamiltonJacobi formulation of the Friedmann equations and does not connect with supersymmetric and supergravity theories [25]. Also the idea to apply system (11,12) instead of the original equations of motion and to seek in such a way exact special solutions is actively used in two-dimensional fields models [26]. In the present work we have shown how to write a first-order formalism to FRW cosmology and to the quintom model of dark energy with two scalar fields. The crucial ingredient was the introduction of a new function, $W = W(\phi, \sigma)$ from which we could express Hubble's parameter in the form $H(t) = W[\phi(t), \sigma(t)]$. Also the energy density and pressure can be obtained by $H(t) = W[\phi(t), \sigma(t)]$. Finally by using the energy density and pressure we obtained the equation of state for the quintom model. The condition for the accelerated expansion is obtained by equation of state. The deformation procedure for two scalar field in quintom model of dark energy may be intersecting for future work.

References

[1] A. G. Riess *et al.* [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998) [astro-ph/9805201];
S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], Astrophys. J. **517**, 565 (1999) [astro-ph/9812133];
P. Astier *et al.* [The SNLS Collaboration], Astron. Astrophys. **447**, 31 (2006) [astro-ph/0510447].

[2] M. Tegmark *et al.* [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [astro-ph/0310723];
K. Abazajian *et al.* [SDSS Collaboration], Astron. J. **128**, 502 (2004) [astro-ph/0403325];
K. Abazajian *et al.* [SDSS Collaboration], Astron. J. **129**, 1755 (2005) [astro-ph/0410239].

[3] D. N. Spergel *et al.* [WMAP Collaboration], Astrophys. J. Suppl. **148**, 175 (2003) [astro-ph/0302209];
D. N. Spergel *et al.* [WMAP Collaboration], astro-ph/0603449.

[4] A. Einstein, Sitzungsber. K. Preuss. Akad. Wiss. 142 (1917) [*The Principle of Relativity* (Dover, New York, 1952), p. 177].

[5] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989);
V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000) [astro-ph/9904398];
S. M. Carroll, Living Rev. Rel. **4**, 1 (2001) [astro-ph/0004075];
P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003) [astro-ph/0207347];
T. Padmanabhan, Phys. Rept. **380**, 235 (2003) [hep-th/0212290].

[6] P. J. Steinhardt, in *Critical Problems in Physics*, edited by V. L. Fitch and D. R. Marlow (Princeton University Press, Princeton, NJ, 1997).

[7] P. J. E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988);
B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988);
C. Wetterich, Nucl. Phys. B **302**, 668 (1988);
R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998) [astro-ph/9708069];
A. R. Liddle and R. J. Scherrer, Phys. Rev. D **59**, 023509 (1999) [astro-ph/9809272];
I. Zlatev, L. M. Wang and P. J. Steinhardt, Phys. Rev. Lett. **82**, 896 (1999) [astro-ph/9807002];
V. Faraoni, Phys. Rev. **D62**, 023504, (2000);
Z. G. Huang, H. Q. Lu, and W. Fang, Class. Quant. Grav. **23**, 6215, (2006), [hep-th/0604160].

[8] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438 (2000) [astro-ph/0004134];
C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. D **63**, 103510 (2001) [astro-ph/0006373];
T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D **62** (2000) 023511.

[9] A. Sen, JHEP **0207**, 065 (2002) [hep-th/0203265];
T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002) [hep-th/0204150];
M. R. Setare, Phys. Lett. B **653**, 116,(2007), arXiv:0705.3517 [hep-th].

[10] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002) [astro-ph/9908168];
R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. **91**, 071301 (2003) [astro-ph/0302506];
S. Nojiri and S. D. Odintsov, Phys. Lett., B **562**, 147 (2003) [hep-th/0303117];
S. Nojiri and S. D. Odintsov, Phys. Lett., B **565**, 1 (2003) [hep-th/0304131];
V. Faraoni, Class. Quant. Grav.22, 3235, (2005);
M. R. Setare, Eur. Phys. J. **C50**, 991, (2007).

[11] F. Piazza and S. Tsujikawa, JCAP **0407**, 004 (2004) [hep-th/0405054].

[12] B. Feng, X. L. Wang, X. Zhang, Phys. Lett. **B607**, 35, (2005); B. Feng, astro-ph/0602156; E. Elizalde, S. Nojiri, S. D. Odintsov, Phys. Rev. **D70**, 043539,(2004); S. Nojiri , S. D. Odintsov, S. Tsujikawa, Phys. Rev. **D71**, 063004, (2005).

[13] L. Amendola, Phys. Rev. D **62**, 043511 (2000) [astro-ph/9908023];
D. Comelli, M. Pietroni and A. Riotto, Phys. Lett. B **571**, 115 (2003) [hep-ph/0302080];
M. R. Setare, Phys. Lett. B **642**, 1, (2006);
M. R. Setare, JCAP **0701**, 023 (2007) [hep-th/0701242].

[14] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B **511**, 265 (2001) [gr-qc/0103004].

[15] M. Li, Phys. Lett. B **603**, 1 (2004); K. Enqvist and M. S. Sloth, Phys. Rev. Lett. **93**, 221302 (2004) [hep-th/0406019];
K. Ke and M. Li, Phys. Lett. B **606**, 173 (2005) [hep-th/0407056];
D. Pavon and W. Zimdahl, Phys. Lett. B **628**, 206 (2005) [gr-qc/0505020];
E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D **71**, 103504 (2005) [hep-th/0502082];
B. Hu and Y. Ling, Phys. Rev. D **73**, 123510 (2006) [hep-th/0601093];
H. Li, Z. K. Guo and Y. Z. Zhang, Int. J. Mod. Phys. D **15**, 869 (2006) [astro-ph/0602521];
M. R. Setare, Phys. Lett. B **644**, 99, (2007)[hep-th/0610190];
M. R. Setare, J. Zhang, X. Zhang, gr-qc/0611084, JCAP **0703**, 007 (2007);
M. R. Setare, Phys. Lett. B **648**, 329, (2007).

[16] X. H. Meng and P. Wang, hep-ph/0311070; V. B. Johri, astro-ph/0311293; M. Sami and A. Toporensky, gr-qc/0312009; M. Szydlowski, W. Czaja and A. Krawiec, astro-ph/0401293; J. Lima and J. S. Alcaniz, astro-ph/0402265; M. Bouhmadi-Lopez and J. J. Madrid, astro-ph/0404540; L. P. Chimento and R. Lazkoz, astro-ph/0405518; Y. H. Wei and Y. Tian, gr-qc/0405038; V. K. Onemli and R. P. Woodard, gr-qc/0406098; P. F. Gonzalez-Diaz and C. L. Siguenza, astro-ph/0407421; P. F. Gonzalez-Diaz, hep-th/0408225; Y. H. Wei, gr-qc/0410050; S. Nojiri and S. D. Odintsov, Phys. Rev. **D72**, 023003, (2005); S. Capozziello, S. Nojiri and S. D. Odintsov, Phys. Lett. **B632**, 597, (2006); S. Nojiri and S. D. Odintsov, hep-th/0506212; E. N. Saridakis, Phys. Lett. **B660**, 138, (2008) ; E. N. Saridakis, 0712.2672 [astro-ph]; E. N. Saridakis, Phys. Lett. **B661**, 335, (2008).

[17] A. Vikman, Phys. Rev. D **71**, 023515 (2005).

[18] Gong-Bo Zhao, Jun-Qing Xia , Mingzhe Li , Bo Feng, Xinmin Zhang , Phys. Rev. **D72**, 123515, (2005).

[19] Z. K. Guo et al, astro-ph/0410654.

[20] Gong-Bo Zhao, Jun-Qing Xia , Bo Feng , Xinmin Zhang , astro-ph/0603621; Jun-Qing Xia, Gong-Bo Zhao, Bo Feng , Xinmin Zhang , astro-ph/0603393; Jun-Qing Xia, Bo Feng, Xin-Min Zhang ,Mod. Phys. Lett. **A20**, 2409, (2005); Bo Feng, Mingzhe Li , Yun-Song Piao , Xinmin Zhang, Phys. Lett. **B634**, 101, (2006); M. R. Setare, Phys. Lett. B **641**, 130 (2006).

[21] V. I. Afonso, D. Bazeia, and L. Losano, Phys. Lett. B **634**, 526, (2006).

[22] D. Bazeia, L. Losano, and J. J. Rodrigues, hep-th/0610028; D. Bazeia, L. Losano, and R. Rosenfeld, astro-ph/0611770.

[23] D. Bazeia, C. B. Gomes, L. Losano, and R. Menezes, Phys. Lett. **B633**, 415, (2006).

[24] J. Sadeghi, A. Mohammadi, Eur. Phys. J. **C49**, 859, (2007).

[25] A. G. Muslimov, Class. Quant. Grav. 7, 231, (1990); D. S. Salopek, J. R. Bond, Phys. Rev. D **42**, 3936, (1990).

[26] D. Bazeia, M. J. dos Santos, R. F. Ribeiro, Phys. Lett. A **208**, 84, (1995); D. Bazeia, F. A. Brito, Phys. Rev. D **61**, 105019, (2000).